**Proof that quadrilaterals are cyclic if and only if their opposite angles are supplementary**

It is easy to show that in a quadrilateral inscribed in a circle, opposite angles are supplementary (they add up to 180 degrees (see Euclid's Elements)). Use the fact that angles subtended by the same arc are equal. The converse is also true: if opposite angles are supplementary then the quadrilateral is cyclic. Consider the quadrilateral ABCD with the circumcircle of triangle ABC. Vertex D cannot be outside or inside the circle because angles ABC and CEA are supplementary, but angles ABC and CDA are also supplementary therefore D has to coincide with E.