

Dynamics of a logistic equation with unbounded time depend coefficients

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Abstract

The aim of this talk is the study of the following non-autonomous logistic ODE

$$\begin{cases} x'(t) = \rho(t)x(t) - \eta(t)x^2(t), & t \in \mathbb{R} \\ x(s) = x_s, & x_s \in \mathbb{R}, \end{cases}$$

where both functions $\rho, \eta : \mathbb{R} \rightarrow \mathbb{R}$ are not necessary bounded, $\eta(t) > 0$ and $\rho(t)$ can be negative in a bounded subset. The main idea in this case is to use the well known techniques in the autonomous case to obtain a function that gives us information about the dynamic of the system in a very simple way. Then we will apply that to a non-autonomous prey-predator system to obtain the permanence of solutions.

References

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