IIMAS SEMINAR

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On the solvability of the equation div $\mathbf{v} = F$

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Abstract

In this talk we present results concerning the solvability of the equation **div** $\mathbf{v} = F$ in various spaces of functions for the vector field $\mathbf{v}$. We find necessary and sufficient conditions on the right hand side F that guarantees the existence of solutions $\mathbf{v}$. We show that the equation **div** $\mathbf{v} = F$ has a solution $\mathbf{v}$ in the space of continuous vector fields vanishing at infinity if and only if F belongs to a closed subspace of the dual of $BV_{\frac{m}{m-1}}(\mathbb{R}^m)$ (where the latter is the space of functions in $L^{\frac{m}{m-1}}(\mathbb{R}^m)$ whose distributional gradient is a vector valued measure). In particular we show that, even though **div** $(\nabla u) = \Delta u = f \in L^m(\mathbb{R}^m)$ need not have a solution $u \in C^1(\mathbb{R}^m)$ to each $f \in L^m(\mathbb{R}^m)$, there corresponds a continuous vector field $\mathbf{v}$ vanishing at infinity such that **div** $\mathbf{v} = f$. This is a joint work with Thierry De Pauw.