Lyapunov-Razumikhin techniques for state-dependent delay differential equations

F.M.G. Magpantay

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Abstract

We present theorems for the Lyapunov and asymptotic stability of the steady state solutions to general state-dependent delay differential equations (DDEs) using Lyapunov-Razumikhin methods. These theorems build upon the previous work of Hale and Verduyn Lunel [2], and Barnea [1] which were mainly aimed at equations with simpler delay terms (e.g. constant and time-dependent delays), and are less applicable to state-dependent DDEs such as the following model equation,

$$\dot{u}(t) = \mu u(t) + \sigma u\big(t - a - cu(t)\big).$$

For fixed a and c, the stability region Σ_* of the zero solution to this model problem is known, and it is the same for both the constant delay ($c = 0$) and state-dependent delay ($c \neq 0$) cases. Using our results we can prove stability in parts of Σ_*, considerably expanding upon the work of Barnea [1] who considered the simpler $\mu = c = 0$ constant delay case. These derived parameter regions contain the entire delay-independent portion of Σ_* and parts of the delay-dependent portion. Similar techniques are also used to derive a condition for global asymptotic stability of the zero solution to the model problem under certain conditions, and to derive bounds on the periodic solutions when the zero solution is unstable.

This is joint work with A.R. Humphries.

References

[1] D.I. Barnea. A method and new results for stability and instability of autonomous functional differential equations. *SIAM J. Appl. Math*, 17(4):681–697, 1969.

[2] J. Hale and S.M. Verduyn Lunel. *Introduction to Functional Differential Equations*, volume 99 of *Applied Mathematical Sciences*. Springer-Verlag, New York, 1993.