Reaction terms avoiding aggregation in slow fluids

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In my talk I will introduce the concepts of chemotaxis and Keller-Segel systems together with its relevance in biology. Then I will focus on some related systems of PDEs arising in biology describing chemotaxis in fluids. This is a research topic bringing the attention of many scientist because it has given rise to many challenging questions having relevance in both biology and mathematics. For example: *Can fluid flows avoid or delay blow-up phenomena happening in Keller-Segel systems?* Such kind of information could give us information about the relevance of chemotaxis in fertilization processes. In my talk, I will present some first insights to attack this kind of problems from the point of view of mathematics. In particular, I will concentrate on the system

$$\begin{aligned} c_t + \mathbf{u}\cdot\nabla c &= \Delta c - a_1 c + \rho, \\ \rho_t + \mathbf{u}\cdot\nabla\rho &= \Delta\rho - \chi\nabla\cdot(\rho\nabla c) - \varepsilon\rho^q, \\ a_2\mathbf{u}_t + \nabla P &= \eta\Delta\mathbf{u} - \rho\nabla\phi, \\ \nabla\cdot\mathbf{u} &= 0. \end{aligned}$$

arising in the modeling of the fertilization process of corals and some other invertebrates. More precisely I will present a new result saying that in case $q \geq 2$, $\Omega \subset \mathbb{R}^2$ bounded, and independently of the size of $|\rho_0|_{L^1}$, there exists global weak solutions. In other words, in this case cells cannot aggregate! However the corresponding case in dimension $n = 3$ remains open. I will discuss about the possibility of having blow-up in this case and higher dimensions.

This is a joint work with Takashi Suzuki (Osaka University, Japan)