4. In Section 1.1, we guessed solutions to the exponential growth model $dP/dt = kP$, where k is a constant (see page 6). Using the fact that this equation is separable, derive these solutions by separating variables.

In Exercises 5–24, find the general solution of the differential equation specified. (You may not be able to reach the ideal answer of an equation with only the dependent variable on the left and only the independent variable on the right, but get as far as you can.)

5. $\dfrac{dy}{dt} = (ty)^2$

6. $\dfrac{dy}{dt} = t^4 y$

7. $\dfrac{dy}{dt} = 2y + 1$

8. $\dfrac{dy}{dt} = 2 - y$

9. $\dfrac{dy}{dt} = e^{-y}$

10. $\dfrac{dx}{dt} = 1 + x^2$

11. $\dfrac{dy}{dt} = 2ty^2 + 3y^2$

12. $\dfrac{dy}{dt} = \dfrac{t}{y}$

13. $\dfrac{dy}{dt} = \dfrac{t}{t^2 y + y}$

14. $\dfrac{dy}{dt} = t\sqrt[3]{y}$

15. $\dfrac{dy}{dt} = \dfrac{1}{2y+1}$

16. $\dfrac{dy}{dt} = \dfrac{2y+1}{t}$

17. $\dfrac{dy}{dt} = y(1-y)$

18. $\dfrac{dy}{dt} = \dfrac{4t}{1+3y^2}$

19. $\dfrac{dv}{dt} = t^2 v - 2 - 2v + t^2$

20. $\dfrac{dy}{dt} = \dfrac{1}{ty + t + y + 1}$

21. $\dfrac{dy}{dt} = \dfrac{e^t y}{1+y^2}$

22. $\dfrac{dy}{dt} = y^2 - 4$

23. $\dfrac{dw}{dt} = \dfrac{w}{t}$

24. $\dfrac{dy}{dx} = \sec y$

In Exercises 25–38, solve the given initial-value problem.

25. $\dfrac{dx}{dt} = -xt, \quad x(0) = 1/\sqrt{\pi}$

26. $\dfrac{dy}{dt} = ty, \quad y(0) = 3$

27. $\dfrac{dy}{dt} = -y^2, \quad y(0) = 1/2$

28. $\dfrac{dy}{dt} = t^2 y^3, \quad y(0) = -1$

29. $\dfrac{dy}{dt} = -y^2, \quad y(0) = 0$

30. $\dfrac{dy}{dt} = \dfrac{t}{y - t^2 y}, \quad y(0) = 4$

31. $\dfrac{dy}{dt} = 2y + 1, \quad y(0) = 3$

32. $\dfrac{dy}{dt} = ty^2 + 2y^2, \quad y(0) = 1$

33. $\dfrac{dx}{dt} = \dfrac{t^2}{x + t^3 x}, \quad x(0) = -2$

34. $\dfrac{dy}{dt} = \dfrac{1-y^2}{y}, \quad y(0) = -2$

35. $\dfrac{dy}{dt} = (y^2+1)t, \quad y(0) = 1$

36. $\dfrac{dy}{dt} = \dfrac{1}{2y+3}, \quad y(0) = 1$

37. $\dfrac{dy}{dt} = 2ty^2 + 3t^2 y^2, \quad y(1) = -1$

38. $\dfrac{dy}{dt} = \dfrac{y^2+5}{y}, \quad y(0) = -2$