Movimento Kepleriano Perturbado

Movimiento Kepleriano

$\Downarrow$

Problema de 2 cuerpos

$\downarrow$

Problema de campo central

- Sistema Solar: Masa del Sol >>> masas de los planetas

Movimiento de los planetas → Problema 2 dos cuerpos (Sol + planeta) + atracción de los otros planetas, lunas, asteroides, comet

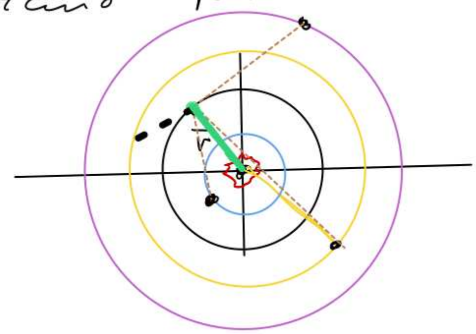
oblatez del Sol

Movimiento kepleriano

$$\ddot{\bar{r}} = -\mu \frac{\bar{r}}{r^3}$$

Movimiento kepleriano perturbado

$$\rightarrow \quad \ddot{\bar{r}} = -\mu \frac{\bar{r}}{r^3} + \bar{F}$$



El planeta va a estarse moviendo sobre orbitas keplerianas

→ orbita kepleriana osculante

Orbitas keplerianas $\Longrightarrow$ Elementos orbitales constantes

$a \quad \epsilon \quad i \quad \Omega \quad \omega \quad T$

orbitas keplerianas perturbadas $\Longrightarrow$ $a(t) \quad \epsilon(t) \quad i(t)$

$\Omega(t) \quad \omega(t) \quad T(t)$

funciones suaves que dependen del tiempo

- orbita kepleriana osculante

Los cambios los vamos a suponer muy pequeños

$$|\mathring{a}(t)| \ll 1 \quad \ldots\ldots$$

$$|\mathring{\epsilon}(t)| \ll 1$$

Pretendemos encontrar
son las EDO que determinan los
cambios de los parámetros orbitales ← perturbación

$$
\begin{array}{l}\text{Ecuaciones} \\ \text{variacio-} \\ \text{nales}\end{array}
\left\{
\begin{aligned}
\frac{da}{dt} &= \tilde{f}_a(t, a, e, i, \Omega, \omega, T, F) \\
\frac{de}{dt} &= \tilde{f}_e(\quad \text{''} \quad) \\
\frac{di}{dt} &= \tilde{f}_i(\quad \text{''} \quad) \\
\frac{d\Omega}{dt} &= \tilde{f}_\Omega(\quad \text{''} \quad) \\
\frac{d\omega}{dt} &= \tilde{f}_\omega(\quad \text{''} \quad) \\
\frac{dT}{dt} &= \tilde{f}_T(\quad \text{''} \quad)
\end{aligned}
\right.
$$

Encontrar las ec. variacionales de
$\bar{c}$, c, $\bar{e}$, e

Momento angular

$$\frac{d}{dt}\left(\bar{c} = \bar{r} \times \bar{v}\right) \longrightarrow \dot{\bar{c}} = \cancel{\dot{\bar{r}} \times \bar{v}} + \bar{r} \times \dot{\bar{v}}$$

$\dot{\bar{r}} = \bar{v}$

$$\dot{\bar{c}} = \bar{r} \times \dot{\bar{v}} = \bar{r} \times \ddot{\bar{r}} = \bar{r} \times \left(-\mu\frac{\bar{r}}{r^3} + \bar{F}\right)$$

$$\dot{\bar{c}} = -\frac{\mu}{r^3}\underbrace{\bar{r} \times \bar{r}}_{=0} + \bar{r} \times \bar{F}$$

$$\boxed{\dot{\bar{c}} = \bar{r} \times \bar{F} \qquad \text{torca}}$$

Encontrar $\bar{e}$

$$\frac{d}{dt}\left(\mu\left(\frac{\bar{r}}{r} + \bar{e}\right) = \bar{V} \times \bar{C}\right)$$

$\dot{\bar{C}} = \bar{r} \times \bar{F}$

$$\mu\left(\frac{d}{dt}\left(\frac{\bar{r}}{r}\right) + \dot{\bar{e}}\right) = \dot{\bar{V}} \times \bar{C} + \bar{V} \times \dot{\bar{C}}$$

$\ddot{\bar{r}} = -\mu\frac{\bar{r}}{r^3} + F$

recordemos $\quad \frac{d}{dt}\left(\frac{\bar{r}}{r}\right) = \frac{\bar{C} \times \bar{r}}{r^3}$

$$\mu\left(\cancel{\frac{\bar{C} \times \bar{r}}{r^3}} + \dot{\bar{e}}\right) = \left(\cancel{-\mu\frac{\bar{r}}{r^3}} + F\right) \times \bar{C} + \bar{V} \times (\bar{r} \times \bar{F})$$

$$\mu\dot{\bar{e}} = \bar{F} \times \bar{C} + \bar{V} \times (\bar{r} \times \bar{F})$$

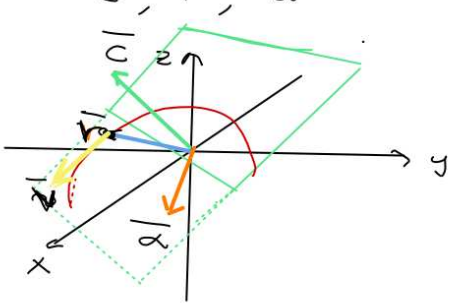
velocidad

Introducimos un sistema coordenado más adecuado

$\bar{c}$, $\bar{r}$ dando $\bar{c} \perp \bar{r}$ $\qquad \bar{c} = \bar{r} \times \bar{v}$

$$\bar{\alpha} = \bar{c} \times \bar{r} \quad \Rightarrow \quad \bar{\alpha} \perp \bar{c} \qquad \bar{\alpha} \perp \bar{r}$$

$\bar{c}$, $\bar{r}$, $\bar{\alpha}$ sistema ortogonal



$\bar{r}$ y $\bar{\alpha}$ están siempre sobre ~~la~~ el plano de la eclíptica

$\bar{c}$ es norma a la eclíptica

sistema coordenado

$(\bar{r}, \bar{c}, \bar{\alpha})$

$$\bar{r} \times \bar{\alpha} = \bar{r} \times (\bar{c} \times \bar{r}) = r^2 \bar{c} \quad \Leftarrow$$

$\bar{a} \times (\bar{b} \times \bar{c}) =$
$(\bar{a} \cdot \bar{c})\, \bar{b} - (\bar{a} \cdot \bar{b})\, \bar{c}$

$$\bar{\alpha} \times \bar{c} = (\bar{c} \times \bar{r}) \times \bar{c} = c^2 \, \bar{r}$$

El vector velocidad siempre está sobre la eclíptica

$\Rightarrow$ $\bar{V}$ se puede expandir como comb. lineal de $\bar{r}$ y $\bar{\alpha}$

$$\bar{V} = A\, \bar{r} + B\, \bar{\alpha} \qquad A, B \text{ escalares}$$

¿cuánto vale A y B?

$$\bar{c} = \bar{r} \times \bar{v} = \bar{r} \times (A\bar{r} + B\bar{\alpha})$$

$$\bar{c} = B\,\bar{r} \times \bar{\alpha} = B\,r^2\,\bar{c}$$

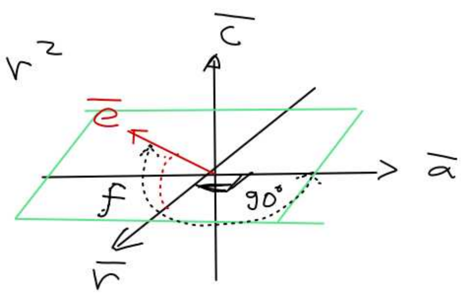
$$\cancel{\bar{c}} = B\,r^2\,\cancel{\bar{c}}$$

$$B = \frac{1}{r^2}$$

$$\bar{e} \cdot \bar{r} = e\,r\cos(f)$$

Cómo determino a A

$$\rightarrow \quad \bar{r} \cdot \bar{v} = \bar{r} \cdot (A\bar{r} + B\bar{\alpha}) = A\,r^2$$



$$\bar{e} \cdot \bar{\alpha} = e\,\alpha\cos(f + 90°)$$

$$\rightarrow \quad \bar{e} \cdot \bar{\alpha} = -e\,\alpha\,\mathrm{sen}(f)$$

Calculemos

$$\bar{e}\cdot\bar{V} = \bar{e}\cdot(A\,\bar{r} + B\,\bar{\alpha})$$

$$= A\,e\,r\cos(f) - \underset{1/r^2}{B}\,e\,c\,r\,\mathrm{sen}(f)$$

$$\bar{e}\cdot\bar{V} = A\,e\,r\cos(f) - \frac{e\,c\,\cancel{r}}{r\cancel{^2}}\,\mathrm{sen}(f) \longleftarrow$$

$$\bar{V}\cdot\left(\mu\left(\frac{\bar{r}}{r} + \bar{e}\right) = \bar{V}\times\bar{c}\right)$$

$$\mu\left(\frac{\bar{V}\cdot\bar{r}}{r} + \bar{e}\cdot\bar{V}\right) = 0 \longrightarrow \underbrace{\bar{V}\cdot\bar{r}}_{A\,r^2} = -r(\bar{e}\cdot\bar{V})$$

$$A\, r^2 = -r \left\{ A\, e\, r \cos(f) - \frac{e\, c}{r} \operatorname{sen}(f) \right\}$$

despejo la A

$$A\, r^2 \,(\underbrace{1 + e \cos f}_{\text{Ec de la cónica}}) = e\, c \operatorname{sen}(f)$$

$$r = \frac{c^2/\mu}{1 + e \cos f}$$

$$A = \mu \frac{e \operatorname{sen}(f)}{r\, c}$$

Ya tenemos a A y a B

$\bar{V} = A \bar{r} + B \bar{\alpha}$