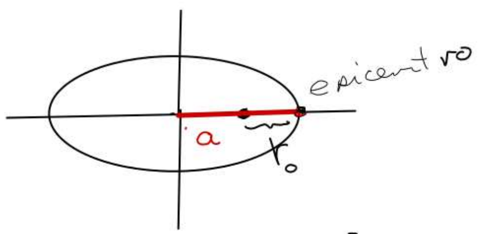
Cálculo de $\dot{a}$ y $\dot{I}$



Cálculo de $\dot{a}$

Exclusivo para el movimiento elíptico

$$0 \leq e < 1$$

$$r = \frac{c^2/\mu}{1 + e\cos(f)}$$

Si $f=0$ estamos en el epicentro

$$r_0 = \frac{c^2/\mu}{1+e}$$

$$(1-e^2)\,a = r_0$$

$$a(1-e^2)\mu = c^2$$

Sabemos que $\dot{c} = r F_\alpha$

$$\mu e \dot{e} = c e F_r \,\mathrm{sen}(f) + \frac{ce\left(e + 2\cos(f) + e\cos^2(f)\right)}{1 + e\cos(f)} F_\alpha$$

$$\frac{d}{dt}\left(a(1-e^2)\mu = c^2 \right)$$ derivamos

$$\dot{a}(1-e^2)\mu - 2ae\dot{e}\mu = 2c\dot{c}$$

$$\dot{a}\frac{c^2}{a} - 2a\left(ce\,F_r \operatorname{sen}(f) + \frac{ce\,(e+2\cos f) + e\cos^2(f)}{1+e\cos(f)}F_\alpha\right) = 2crF_\alpha$$

$$\dot{a} = \frac{2a^2e}{c}\operatorname{sen}(f)F_r + \left(\frac{2a^2e}{c}\,\frac{(e+2\cos(f)+e\cos^2(f))}{1+e\cos(f)} + \frac{2ar}{c}\right)F_\alpha$$

$$\dot{a} = \frac{2a^2e}{c}\operatorname{sen}(f)F_r + \frac{2a^2c}{\mu r}\left\{\frac{e\mu r(e+2\cos(f)+e\cos^2(f))}{c^2\,(1+e\,(f))} + \frac{r^2\mu}{a c^2}\right\}F_\alpha$$

$$\dot{a} = \frac{2a^2e}{c}\operatorname{sen}(f)F_r + \frac{2a^2c}{\mu r}\left\{\frac{e}{c^2}\mu r\left(\frac{\mu}{c^2}r\right)(e+2\cos(f)+e\cos^2(f)) + \frac{r^2\mu}{c^2}\,\frac{(1-e^2)\mu}{c^2}\right\}F_\alpha$$

$$\dot{a} = \frac{2a^2 e}{c} \operatorname{sen}(f) F_r + \frac{2a^2 c}{\mu r} \left\{ \frac{\mu^2 r^2}{c^4} \left[\cancel{e^2} + \underbrace{2e\cos(f) + e^2\cos^2(f) + 1}_{} - \cancel{e^2} \right] \right\} F_\alpha$$

$$\dot{a} = \frac{2a^2 e}{c} \operatorname{sen}(f) F_r + \frac{2a^2 c}{\mu r} \left\{ \frac{\mu^2 r^2}{c^4} \underbrace{(1 + e\cos(f))^2}_{} \right\} F_\alpha$$

$$\dot{a} = \frac{2a^2 e}{c} \operatorname{sen}(f) F_r + \frac{2a^2 c}{\mu r} \left\{ \cancel{\frac{\mu^2 r^2}{c^4}} \left(\cancel{\frac{c^2}{\mu r^2}} \right)^2 \right\} F_\alpha$$

$$\boxed{\dot{a} = \frac{2a^2 e}{c} \operatorname{sen}(f) F_r + \frac{2a^2 c}{\mu r} F_\alpha}$$

Cálculo de $\dot{T}$

3er Ley $\frac{d}{dt}\left(n = \underbrace{\frac{\sqrt{\mu}}{a^{3/2}}} \right)$

$$\dot{n} = \sqrt{\mu}\left(-\frac{3}{2}\right) a^{-5/2}\,\dot{a} = \sqrt{\mu}\left(-\frac{3}{2}\right) a^{-3/2}\, a\,\dot{a}$$

$$\dot{n} = -\frac{3}{2}\, n\, a\, \dot{a}$$

Para calcular a $\dot{T}$ usamos las Ec. de Kepler

$$r = a(1 - e\cos(u))$$

$$n(t - T) = u - e\,\mathrm{sen}(u)$$

$$\bar{r}\cdot\bar{v} = \bar{r}\cdot\dot{\bar{r}} = r\dot{r} = r\frac{dr}{du}\frac{du}{dt} = \cancel{r}\frac{dr}{du}\frac{k}{\cancel{r}} = \sqrt{\frac{\mu}{a}}\frac{dr}{du} = \sqrt{\frac{\mu}{a}}\, a\, e\,\mathrm{sen}(u)$$

$$r\dot{r} = \sqrt{\mu a}\ e \operatorname{sen}(u)$$

$$\frac{d}{dt}\left(r = a(1 - e\cos(u)) \right)$$

$$\dot{r} = \dot{a}\,(1 - e\cos(u)) + a e \operatorname{sen}(u)\,\dot{u} - a\dot{e}\cos(u)$$

$$\frac{\sqrt{\mu a}}{r}\, e \operatorname{sen}(u) = \dot{a}\,(1 - e\cos(u)) + a\left(e\dot{u} \operatorname{sen}(u) - \dot{e}\cos(u) \right)$$

$$\frac{d}{dt}\left(n(t - T) = u - e\operatorname{sen}(u) \right)$$

$$\dot{n}(t - T) + n(1 - \dot{T}) = \dot{u} - \dot{e}\operatorname{sen}(u) - e\cos(u)\,\dot{u}$$

$$= \dot{u}(1 - e\cos(u)) - e\operatorname{sen}(u)$$

$$\frac{\sqrt{\mu a}}{r}\, e \operatorname{sen}(u) = \dot a\,(1 - e\cos(u)) + a\left(e\operatorname{sen}(u)\left\{\frac{\dot n(t-T) + n(1-\dot T) + \dot e \operatorname{sen}(u)}{1 - e\cos(u)}\right\} - \dot e \cos u\right)$$

$$\dot n = -\frac{3}{2}\, n\, a\, \dot a \qquad 1 - e\cos u = \frac{r}{a} \qquad r^2 \dot f = C$$

velocidad aereolar $\quad r^2 \dot f = \sqrt{\mu a (1-e^2)}$

$$\frac{d}{dt}\left(r = \frac{a(1-e^2)}{1 + e\cos(f)}\right) \rightarrow \dot r = \frac{a(1-e^2)}{1+e\cos(f)}\, e \operatorname{sen}(f)\, \dot f = \underbrace{\frac{a(1-e^2)\, r^2 e \operatorname{sen}(f)}{a^2(1-e^2)^2}\, \frac{\sqrt{\mu a(1-e^2)}}{r^2}}$$

$$\underbrace{\dot r = \frac{\sqrt{\mu a}}{r}\, e \operatorname{sen}(u) = \frac{\sqrt{\mu}\, e \operatorname{sen}(f)}{\sqrt{a(1-e^2)}}}$$

$$\operatorname{sen}(u) = \frac{r \operatorname{sen}(f)}{a\sqrt{1-e^2}}$$

$$\dot{T}\, n\, e\, r \operatorname{sen}(f) = \dot{a}\left\{ \frac{r^2\sqrt{1-e^2}}{a^2} - \frac{3}{2}\, e\, r \operatorname{sen}(f)\, n\, a\,(t-T) \right.$$

$$\left. + \dot{e}\left\{ \frac{e^2 r^2 (1-\cos^2(f))}{e\, a\sqrt{1-e^2}} - \frac{a(1-e^2)}{e\, a\sqrt{1-e^2}}\left(r - \frac{r^2}{a}\right) \right\} \right.$$

Así tenemos ya nuestras ecuaciones para

$\dot{a}$, $\dot{e}$, $\frac{di}{dt}$, $\dot{\omega}$, $\dot{\Omega}$, $\dot{T}$

en función de $r(t)$, $f(t)$ y las fuerzas perturbativas F_α, F_r y F_c