Parámetros orbitales perturbados

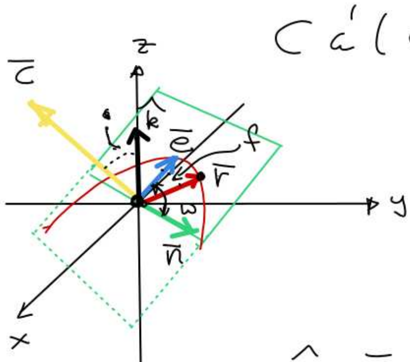
$$\dot{\Omega} = \frac{r F_c \,\mathrm{sen}(\omega + f)}{c\,\mathrm{sen}(i)}$$

$$\mu \dot{e} = c e F_r \,\mathrm{sen}(f) + c e F_a \left\{ \frac{e + 2\cos(f) + e\cos^2(f)}{1 + e\cos(f)} \right\}$$

Cálculo de $\dot{\omega}$

$\longrightarrow \bar{n} = \hat{k} \times \bar{c}$ $\qquad n = c\,\mathrm{sen}(i)$

$a \times b \times c = (a \cdot c)\, b - (a \cdot b)\, c$



$$\underbrace{\bar{n} \times \bar{e}}_{\text{magnitud}} = (\hat{k} \times \bar{c}) \times \bar{c}$$

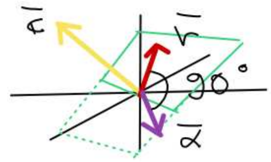
$$= (\hat{k} \cdot \bar{e})\, \bar{c} - \underbrace{(\bar{c} \cdot \bar{e})}_{=0}\, \hat{k}$$

$n\, e\, \mathrm{sen}(\omega) = (\hat{k} \cdot \bar{e})\, c$ $\qquad$ despejamos $\hat{k} \cdot \bar{e}$

✓ $\hat{k} \cdot \bar{e} = \dfrac{n e}{c}\,\mathrm{sen}(\omega) = \dfrac{c\,\mathrm{sen}(i)\, e}{c}\,\mathrm{sen}(\omega)$

$\longrightarrow \hat{k} \cdot \bar{e} = e\,\mathrm{sen}(i)\,\mathrm{sen}(\omega)$

Calculamos $\hat{k}\cdot\bar{r}$



$a \cdot b\times c =$
$\quad -b \cdot a\times c$

$\alpha = c\,r$

$n = c\,\mathrm{sen}(i)$

$\bar{r} = (\bar{\alpha}\times\bar{c})/c^2$

$\hat{k}\cdot\bar{\alpha} = c\,r\,\mathrm{sen}(i)\cos(\omega+f)$

$$\hat{k}\cdot\bar{r} = \hat{k}\cdot\left(\frac{\bar{\alpha}\times\bar{c}}{c^2}\right) = \frac{1}{c^2}\left(-\bar{\alpha}\cdot\hat{k}\times\bar{c}\right) = -\frac{1}{c^2}\,\bar{\alpha}\cdot\bar{n} =$$

$$= -\frac{1}{c^2}\,\alpha\, n\cos(\omega+f+90°)$$

$$\hat{k}\cdot\bar{r} = -\frac{r}{\cancel{c^2}}\cancel{c}\,\alpha\, n\;\mathrm{sen}(i)\cos(\omega+f+90°)$$

$$r\;\hat{k}\cdot\bar{r} = r\,\mathrm{sen}(i)\,\mathrm{sen}(\omega+f)$$

derivemos $\hat{k}\cdot\bar{e}$

$$\frac{d}{dt}\left(\hat{k}\cdot\bar{e} = e\,\mathrm{sen}(i)\,\mathrm{sen}(\omega)\right)$$

$$\hat{k}\cdot\dot{\bar{e}} = \dot{e}\,\mathrm{sen}(i)\,\mathrm{sen}(\omega) + e\cos(i)\,\mathrm{sen}(\omega)\frac{di}{dt} + e\,\mathrm{sen}(i)\cos(\omega)\,\dot{\omega}$$

Por otro lado

$$\hat{k}\cdot\left(\mu\dot{\bar{e}} = \frac{2cF_\alpha}{r}\bar{r} - \left(\frac{F_r}{r} + \frac{Ar}{c}F_\alpha\right)\bar{\alpha} - \frac{Ar^2}{c}F_c\,\bar{c}\right)$$

$$\mu\hat{k}\cdot\dot{\bar{e}} = \frac{2cF_\alpha}{r}\hat{k}\cdot\bar{r} - \left(\frac{F_r}{r} + \frac{Ar}{c}F_\alpha\right)\bar{\alpha}\cdot\hat{k} - \frac{Ar^2}{c}F_c\,\bar{c}\cdot\hat{k}$$

$$= \frac{2cF_\alpha}{r}\,r\,\mathrm{sen}(i)\,\mathrm{sen}(\omega+f) - \left(\frac{F_r}{r} + \frac{Ar}{c}F_\alpha\right)c\,r\,\mathrm{sen}(i)\cos(\omega+f)$$
$$- \frac{Ar^2}{c}F_c\,c\cos(i)$$

subtituyendo y despejando a $\dot{\omega}$

$$\dot{\omega}\left(e\,\mathrm{sen}(i)\cos(\omega)\right) = -\frac{c}{\mu}\left[F_r\,\mathrm{sen}(f) + \frac{F_\alpha\left(e+2\cos(f)+e\cos^2(f)\right)}{1+e\cos(f)}\right]\mathrm{sen}(i)\,\mathrm{sen}(\omega)$$

$$-\,e\cos(i)\underbrace{\frac{r}{c}F_c\cos(\omega+f)}\,\mathrm{sen}(\omega)$$

$$+\frac{1}{\mu}\left[2cF_\alpha\,\mathrm{sen}(i)\,\mathrm{sen}(\omega+f) - \left(\frac{F_r}{r}+\frac{Ar}{c}F_\alpha\right)c\,r\,\mathrm{sen}(i)\cos(\omega+f)\right.$$
$$\left. -\,A\,r\,F_c\cos(i)\right]$$

$$\dot{\omega} = F_r \left\{ \frac{-\frac{c}{\mu} \operatorname{sen}(f) \operatorname{sen}(i) \operatorname{sen}(\omega) - \frac{1}{\mu} c \operatorname{sen}(i) \cos(\omega + f)}{e \operatorname{sen}(i) \cos(\omega)} \right\}$$

$$+ F_c \left\{ \frac{-\frac{e r}{c} \cos(\omega + f) \cos(i) \operatorname{sen}(\omega) - A r \cos(i) / \mu}{e \operatorname{sen}(i) \cos(\omega)} \right\}$$

$$+ F_\alpha \left\{ \frac{-\frac{c}{\mu} \left(e + 2\cos(f) + e \cos^2(f) \right) \operatorname{sen}(i) \operatorname{sen}(\omega)}{\left(1 + e \cos(f)\right) \left(e \operatorname{sen}(i) \operatorname{sen}(\omega)\right)} + \frac{1}{\mu} \frac{2 c \operatorname{sen}(i) \operatorname{sen}(\omega + f) - \frac{A r^2}{c} \operatorname{sen}(i) \cos(\omega + f)}{e \operatorname{sen}(i) \operatorname{sen}(\omega)} \right\}$$