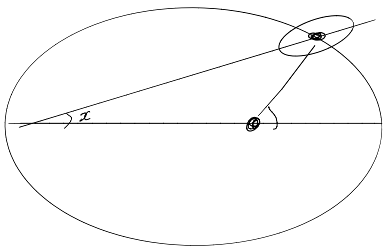
$$H(y,x,t) = \frac{y^2}{2} - \frac{\varepsilon}{2}\left(\frac{a}{r}\right)^3 \cos(2x-2f)$$

- Este depende del tiempo. ↓
- La dependencia es periódica.

$r(t)$, $f(t)$



- Sección de Poincaré
- $\mathcal{L}_X \Omega = 0$

Espacio fase

$x \in \mathbb{T}$, $y \in \mathbb{R}$, $t \in \mathbb{T}$

(y, x) espacio fase $\quad \mathbb{R}\times\mathbb{T}$ cilindro

($\ldots \mathbb{T}$ tiempo)

$\alpha = y\,dx$, $\Omega = d\alpha = dy \wedge dx$

$$\alpha = -dH = -y\,dy - \varepsilon\left(\frac{a}{r}\right)^3 \sin(2x-2f)\,dx$$

$$X = A\frac{\partial}{\partial x} + B\frac{\partial}{\partial y}$$

$$\alpha = i_X \Omega$$

$$= dy\left(A\frac{\partial}{\partial x} + B\frac{\partial}{\partial y}\right)dx - dx\left(A\frac{\partial}{\partial x} + B\frac{\partial}{\partial y}\right)dy$$

$$= B\,dx - A\,dy = -dH$$

$$B = -\varepsilon\left(\frac{a}{r}\right)^3 \sin(2x-2f)$$

$$A = y$$

$$\dot{x} = y$$

$$\dot{y} = -\varepsilon\left(\frac{a}{r(t)}\right)^3 \sin(2x-2f(t))$$

$$d(i_X\Omega) = d\left(-\varepsilon\left(\frac{a}{r}\right)^3 \sin(2x-2f)\,dx - y\,dy\right)$$

$$= 0 = d(i_X\Omega) + i_X \cancel{d\Omega}^{0} = \mathcal{L}_X\Omega$$

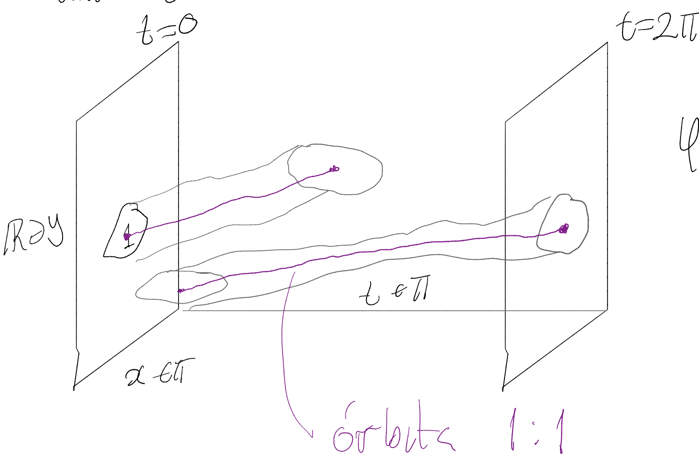
$$\mathcal{L}_X \Omega = 0$$

$$\Rightarrow \frac{d}{dt} \varphi_t^* \Omega = 0$$

$$\Rightarrow \varphi_t^* \Omega = \Omega \quad \forall\, t \in \mathbb{T}$$

$r(t)$ y $f(t)$ son 2π periódicas.

Una sección de Poincaré (un mapeo estroboscópico)



$$\varphi_{2\pi}^* \Omega = \Omega$$

Esto es un problema.

Hay muchos satélites (y ¡mercurio!) que están en resonancia 1:1.

$\mathcal{L}_X \Omega = 0$ es imposible que la órbita sea un atractor.

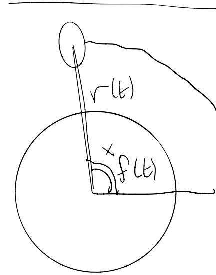
[78] P. Goldreich, Final spin states of planets and satellites, *Astronom. J.* **71**, no. 1, 1–7 (1966)

[79] P. Goldreich, S. Peale, Spin–orbit coupling in the solar system, *Astronom. J.* **71**, no. 6, 425–438 (1966)

[123] G.J.F. MacDonald, Tidal friction, *Rev. Geophys.* **2**, 467–541 (1964)

[147] S.J. Peale, The free precession and libration of Mercury, *Icarus*, **178**, 4–18 (2005)

Torcas de marea



$$J(\dot{x}, t) = -K_d[L(e,t)\dot{x} - N(e,t)]$$

$$L(e,t) = \frac{a^6}{r^6(t)} \qquad N(e,t) = \frac{a^6}{r^6(t)} \dot{f}(t)$$

$$K_d = 3 l_0 \frac{k_2}{\xi Q}\left(\frac{R_e}{a}\right)^3 \frac{M}{m}$$

k_2 - numero de love
Q - factor de calidad
ξ - constante de estructura
$I_3 = \xi m R_e^2$
R_e - radio ecuatorial
M - masa de la Tierra
m - masa de la Luna

$K_d \sim 10^{-8}$
para la Luna y Mercurio

Modelo de órbita-espín con torcas de marea.

$$\dot{x} = y$$
$$\dot{y} = -\varepsilon\left(\frac{a}{r}\right)^3 \sin(2x - 2f) - K_d[L(e,t)y - N(e,t)]$$

$$\mathcal{L}_X \Omega = d(i_X \Omega)$$
$$= d(dy(X)dx - dx(X)dy)$$

$$\boxed{X = X_H - K_d[L(e,t)y - N(e,t)]\frac{\partial}{\partial y}}$$

$$= d\Big(dy\big(X_H - K_d[L(e,t)y - N(e,t)]\tfrac{\partial}{\partial y}\big)dx$$
$$- dx\big(X_H - K_d[L(e,t)y - N(e,t)]\tfrac{\partial}{\partial y}\big)dy\Big)$$
$$= d(\Omega(X_H, \cdot))$$
$$- d(K_d[L(e,t)y - N(e,t)]dx)$$
$$= 0 - K_d[L(e,t)]\, dy \wedge dx$$

$$\mathcal{L}_X \Omega = -K_d[L(e,t)]\Omega \neq 0$$
$$\hookrightarrow X_H - K_d[\quad]\frac{\partial}{\partial y}$$
$$\varphi_t^* \mathcal{L}_X \Omega = -K_d[L(e,t)]\varphi_t^* \Omega$$
$$\frac{d}{dt}\varphi_t^* \Omega = -K_d[L(e,t)]\varphi_t^* \Omega$$
$$\varphi_0^* \Omega = \Omega$$

PVI para $\varphi_t^* \Omega$

EDO.

$$\frac{\frac{d}{dt}\varphi_t^* \Omega}{\varphi_t^* \Omega} = -K_d[L(e,t)]$$

$$\frac{d}{dt}\ln(\varphi_t^* \Omega) = -K_d[L(e,t)]$$

$$\ln(\varphi_t^* \Omega) - \ln(\varphi_0^* \Omega) = -K_d \int_0^t L(e,s)\,ds$$

Ω.

$$\varphi_t^* \Omega = e^{-K_d \int_0^t L(e,s)ds}\,\Omega$$

Para $t = 2\pi$ $\quad \varphi_{2\pi}(y, x)$

$$\varphi_{2\pi}^{*}\,\Omega = e^{-K_d \int_0^{2\pi} L(e,s)\,ds}\,\Omega$$

$$e^{-K_d \int ds} < 1$$

$$\varphi_{2\pi}^{*}\,\Omega = \lambda\,\Omega \,, \quad \lambda < 1.$$

¡Este sistema tiene atractores!