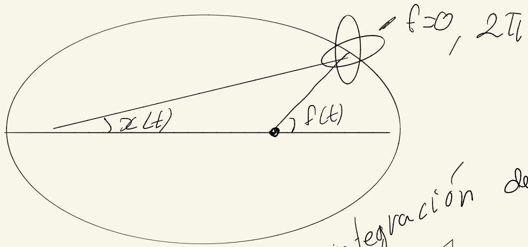
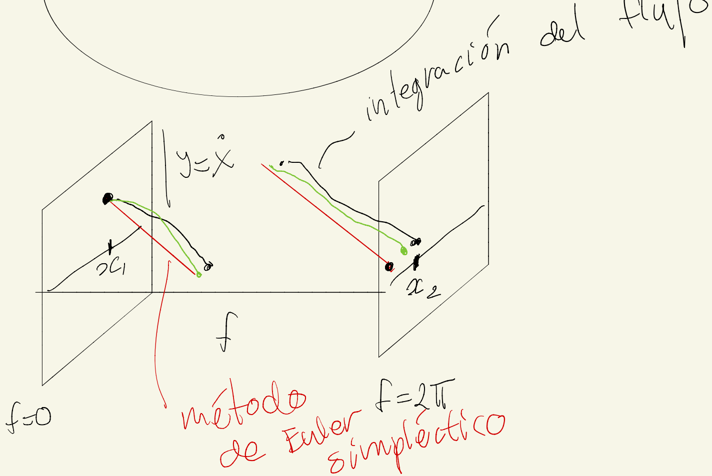
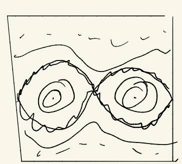
$$\ddot{x} + \varepsilon\left(\frac{a}{r(t)}\right)^{3} \sin(2\pi(2X(t)-2f(t)))$$

$$\sin(2X(t)-2f(t)) = K_d(I(e)\dot{x} - N(e))$$

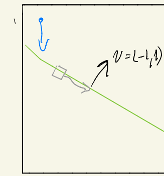




$f(x) = \frac{1}{2\pi}\sin(4\pi x)$, $f(x) = \sin(2x)$



Ejemplo de contornos



(I) $m\ddot{q} = F$

$F = (0, -mg)$

(II) contorno

$y + x = 0$

$$F = F_T + F_N \quad (\lambda \nabla f)$$

1) Escribimos un campo vectorial normal al contorno

(II) $f(x,y) = 0$, $f(x,y) = y + x$

$v = (-1, 1)$ campo constante en $\mathbb{R}^2$

~~$(x^2+y^2+1)(-1,1)$~~

$$\langle m\ddot{q} - F, v\rangle = 0$$

$$\Leftrightarrow \langle (m\ddot{x}, m\ddot{y} + mg), (-1,1)\rangle = 0$$

$$\Leftrightarrow -m\ddot{x} + m\ddot{y} + mg = 0$$

$$\Leftrightarrow -\ddot{x} + \ddot{y} + g = 0 \quad \ldots \quad (A)$$

2) Utilizar las ecuaciones del contorno.

$y(t) + x(t) = 0$, $f(x(t), y(t)) = 0$

$$\ddot{y} + \ddot{x} = 0 \Rightarrow \ddot{y} = -\ddot{x} \qquad (B)$$

$$(A) \Rightarrow \ddot{y} + \ddot{y} + g = 0 \Rightarrow 2\ddot{y} + g = 0 \Rightarrow \ddot{y} = -\frac{g}{2}$$

$$\ddot{x} = \frac{g}{2}$$

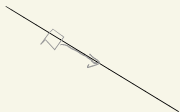
$\ddot{x} = \frac{1}{2}g$

$\ddot{y} = -\frac{1}{2}g$

sist de segundo orden.

$$\ddot{y} = -\frac{1}{2}g \Rightarrow \dot{y} = -\frac{1}{2}gt - B \Rightarrow y(t) = -\frac{1}{4}gt^2 - Bt + A$$

$$x(t) = \frac{1}{4}gt^2 + Bt - A$$



3) $F_N \sim \lambda \nabla f \qquad f(x,y) = y + x$

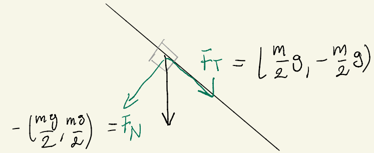
$\nabla f = (1,1) \qquad |\nabla f|^2 = 2$

$\langle m\ddot{q} - F_T - \lambda \nabla f, v \rangle = 0$, $\langle m\ddot{q} - F_T - \lambda\nabla f, \nabla f \rangle = 0$

$\lambda = \frac{1}{|\nabla f|^2}\langle m\ddot{q} - F, v\rangle$, $\lambda = \frac{1}{|\nabla f|^2}\langle m\ddot{q} - F, \nabla f\rangle$

$$= \frac{1}{2}\left\langle \left(\frac{mg}{2}, -\frac{m}{2}g + mg\right), (1,1)\right\rangle$$

$$= \frac{1}{2}\left(\cancel{m\frac{g}{2}} - \cancel{\frac{m}{2}g} + mg\right) = \frac{m}{2}g$$



$$F = (0, -mg) = F_T + F_N = \left(\frac{m}{2}g, -\frac{m}{2}g\right) - \frac{m}{2}g(1,1)$$

Problema restringido al contorno.

$$m\ddot{q} = \left(\frac{m}{2}g, -\frac{m}{2}g\right)$$

Lo que estamos haciendo aquí es reescribir el problema sobre la restricción.

Ponemos una coordenada en términos de las demás.

$$f(x,y,t) = 0 \overset{TFI}{\Rightarrow} f(x, h(x,t), t) = 0$$

$$y = h(x,t)$$

$$\Rightarrow \dot{y} = \tilde{h}(x, \dot{x}, t), \quad \ddot{y} = \tilde{\tilde{h}}(x, \dot{x}, \ddot{x}, t)$$

$$\ddot{x} = \tilde{f}(x, \dot{x}, y, \dot{y}, t) = \tilde{f}(x, \dot{x}, h(x,t), \tilde{h}(x, \dot{x}, t))$$

Ec. diferencial de segundo orden.

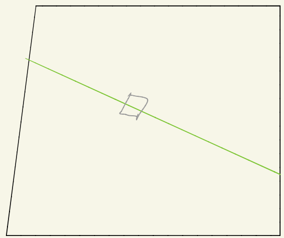
Ejemplo con tiempo

$$f(x,y,t) = y + (1+\varepsilon \sin \omega t)x \quad , \quad \varepsilon \geq 0 \text{ pequeña.}$$

$$f(x,y,t) = 0$$

$$\nabla f(x,y,t)$$

$$\lambda = \lambda(t)$$



$$y = h(x,t) \quad \text{TFI}$$

$$y = -(1+\varepsilon \sin \omega t)x$$

$$\dot{y} = \tilde{h}(x,\dot{x},t)$$

$$\ddot{y} = \tilde{\tilde{h}}(x,\dot{x},\ddot{x},t)$$

$$\ddot{x} = \tilde{f}(\quad \ldots \quad)$$