Dem (Noether)

Sea $q(t)$ sol del sistema

$$\frac{d}{dt}\left[\frac{\partial \mathcal{L}}{\partial v_i}(q(t),\dot q(t))\right] - \frac{\partial \mathcal{L}}{\partial q_i}(q(t),\dot q(t)) = 0 \quad (*)$$

$\forall t \in [t_1, t_2]$

$$\mathcal{L}(q,v) = \tilde{\mathcal{L}}_\varepsilon(q,v), \quad \forall \varepsilon$$

$$= \mathcal{L}(Q_\varepsilon(q), \dot Q_\varepsilon(q))$$

$$\Rightarrow \frac{d}{dt}\left[\frac{\partial \tilde{\mathcal{L}}_\varepsilon}{\partial v_i}(q(t),\dot q(t))\right] - \frac{\partial \tilde{\mathcal{L}}_\varepsilon}{\partial q_i}(\quad) = 0$$

$\forall \varepsilon$

$Q_\varepsilon(q)$ difeo

$$\dot Q_\varepsilon(q)\cdot \dot q = DQ_\varepsilon(q)\cdot \dot q$$

$$\underbrace{\frac{d}{dt}\frac{\partial \mathcal{L}}{\partial \dot Q_{\varepsilon,j}} - \frac{\partial \mathcal{L}}{\partial Q_{\varepsilon,j}} = 0}_{1} \quad \xleftarrow{j=1,\dots,n} \quad \underbrace{\frac{d}{dt}\frac{\partial \tilde{\mathcal{L}}_\varepsilon}{\partial \dot q_j} - \frac{\partial \tilde{\mathcal{L}}_\varepsilon}{\partial q_j} = 0}_{2}$$

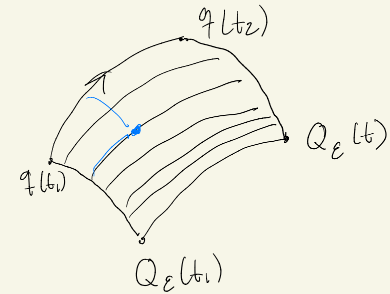
Como $\mathcal{L}$ es invariante $\tilde{\mathcal{L}}_\varepsilon(q,\dot q) = \mathcal{L}(q,\dot q)$

$$2 \Leftrightarrow \frac{d}{dt}\frac{\partial \mathcal{L}}{\partial \dot q} - \frac{\partial \mathcal{L}}{\partial q_j} = 0, \quad j = 1,\dots,n$$

$(*) \Rightarrow 1$ La curva $Q_\varepsilon(t) = Q_\varepsilon(q(t))$

1 es $$\frac{d}{dt}\left[\frac{\partial \mathcal{L}}{\partial v_i}(Q_\varepsilon(t), \dot Q_\varepsilon(t)) - \frac{\partial \mathcal{L}}{\partial q_i}(Q_\varepsilon(t), \dot Q_\varepsilon(t))\right] = 0$$

$\forall \varepsilon$



superficie parametrizada por trayectorias en M.

Superficie de solns de ec. E-L en TM

$$z(t) = [q(t), \dot q(t)]$$

$$\tilde Q_\varepsilon(z(t)) = [Q_\varepsilon(q(t)), [DQ_\varepsilon(q(t))]\cdot \dot q(t)]$$

$$= [Q_\varepsilon(t), \dot Q_\varepsilon(t)]$$

Sea $\Phi : [t_1, t_2] \times \mathbb{R} \longrightarrow M$

$$(t, \varepsilon) \xmapsto{\ \Phi\ } Q_\varepsilon(q(t))$$

$$\Phi(t, \varepsilon)$$

$$\tilde{\Phi}(t, \varepsilon) = [\Phi(t, \varepsilon), \dot{\Phi}(t, \varepsilon)]$$

$$\dot{\Phi}(t, \varepsilon) = \frac{d}{dt} Q_\varepsilon(q(t)) = \frac{\partial}{\partial t} \Phi(t, \varepsilon)$$

(I) Invarianza de $\mathcal{L}$ $\forall t$

$$\frac{\partial}{\partial \varepsilon} \mathcal{L}(\Phi, \dot{\Phi}) = 0 \quad \left(\forall_\varepsilon \ \tilde{\mathcal{L}}_\varepsilon(q, v) = \mathcal{L}(q, v)\right)$$

$$\sum_{i=1}^{n} \left[\frac{\partial \mathcal{L}}{\partial q_i} \cdot \frac{\partial \Phi_i}{\partial \varepsilon} + \frac{\partial \mathcal{L}}{\partial v_i} \cdot \frac{\partial \dot{\Phi}}{\partial \varepsilon} \right] = 0 \quad (A)$$

(II) Sabemos que $\forall \varepsilon$ $\Phi(t, \varepsilon)$, satisface E-L:

$$\frac{d}{dt}\left[\frac{\partial \mathcal{L}}{\partial v_i}(\Phi, \dot{\Phi})\right] = \frac{\partial \mathcal{L}}{\partial q_i}(\Phi, \dot{\Phi}), \quad \forall i \ (B)$$

Usando (B) en (A) tenemos

$$\sum_{i=1}^{n} \underbrace{\left[\frac{d}{dt} \frac{\partial \mathcal{L}}{\partial v_i}(\Phi, \dot{\Phi}) \right]}_{3} \cdot \underbrace{\frac{\partial \Phi_i}{\partial \varepsilon}}_{1} + \frac{\partial \mathcal{L}}{\partial v_i} \underbrace{\frac{\partial \dot{\Phi}}{\partial \varepsilon}}_{2} = 0 \quad (C)$$

1 $= \frac{\partial}{\partial \varepsilon} Q_\varepsilon(q(t))$

2 $= \frac{\partial}{\partial \varepsilon} \frac{\partial}{\partial t} \Phi(t, \varepsilon) = \frac{\partial}{\partial t} \frac{\partial}{\partial \varepsilon} \Phi(t, \varepsilon) = \frac{\partial}{\partial t} \frac{\partial}{\partial \varepsilon} Q_\varepsilon(q(t))$

3 $= \frac{d}{dt} \frac{\partial \mathcal{L}}{\partial v_i}\left(Q_\varepsilon(q(t)), \dot{Q}_\varepsilon(q(t))\right)$

(C) $\Rightarrow$ $\forall \varepsilon \in \mathbb{R}$.

Regla del producto.

$$\frac{d}{dt}\left[\sum_{i=1}^{n} \frac{\partial \mathcal{L}}{\partial v_i}(Q_\varepsilon(q(t)), \dot{Q}_\varepsilon(q(t))) \left(\frac{\partial Q_\varepsilon}{\partial \varepsilon}(q(t))\right)_i \right] = 0$$

$\varepsilon = 0$ esta es la J

$$\frac{d}{dt} J(q(t), \dot{q}(t)) = 0$$

¡Es una cantidad conservada! //