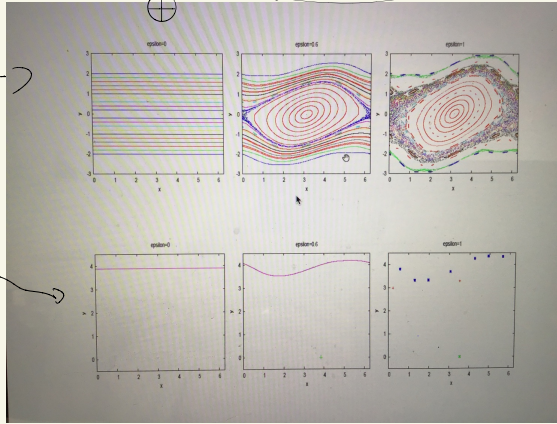


Caso conservativo

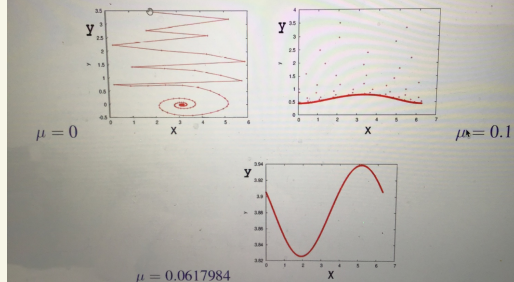
Caso disipativo

No hay atractores.



En el caso disipativo sólo se tiene un toro invariante.

Looking for the torus with frequency $\omega = 2\pi \frac{\sqrt{5}-1}{2} \approx 3.8832$, dissipative standard map with $\varepsilon = 0.1, \lambda = 0.9$.



Este es un buen ejercicio!

Modeling the spin-orbit problem

- Spin-orbit problem:
 - ▷ triaxial satellite \mathcal{S} (with $I_1 < I_2 < I_3$);
 - ▷ center of the satellite moves on a **Keplerian orbit** around a central planet \mathcal{P} ;
 - ▷ spin-axis **perpendicular** to orbit plane and coinciding with **shortest physical axis**;
 - ▷ **NO tidal torque**, due to the non-rigidity.

• Conservative case: equation of motion:

$$\ddot{x} + \varepsilon \left(\frac{a}{r(t)} \right)^3 \sin(2x - 2f(t)) = 0, \quad \varepsilon = \frac{3I_2 - I_1}{2I_3}$$

corresponding to a 1-dim, time-dependent Hamiltonian:

$$\mathcal{H}(y, x, t) = \frac{y^2}{2} - \frac{\varepsilon}{2} \left(\frac{a}{r(t)} \right)^3 \cos(2x - 2f(t)).$$

Spin-orbit problem

- $\gamma_g^\pm = \frac{\sqrt{5} \pm 1}{2}$
- Diophantine frequency: $\omega = 1 + \frac{1}{2 + \gamma_g^-}$ or $\omega = \gamma_g^+$.

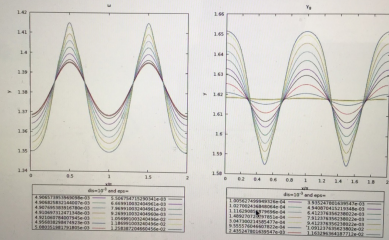
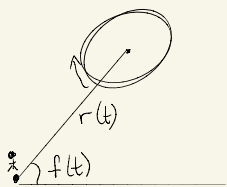


Figure: The invariant attractor for SO with $\eta = 10^{-3}$. Left: ω . Right: γ_g^+ .

Modelo disipativo



P. Goldreich, "Final spin states of planets and satellites," *Astronom. J.*, 71, no. 6. 425-438 (1966)

S. J. Peale, "Icarus 78 4-18 (2005)"

Torcas de marea

$$\tau(\dot{x}, t) = -K_d [L(e, t)\dot{x} - N(e, t)]$$

$$L(e, t) = \frac{a^6}{r(t)^6}, \quad N(e, t) = \frac{a^6}{r(t)^6} \dot{f}(t)$$

K_d - constante disipativa

$$K_d = 3n \frac{k_e}{a} \left(\frac{R_e}{a} \right)^3 \frac{M}{M} \frac{1}{M}$$

k_e - número de Love (constante elástica) $\frac{M}{M}$ - masa de P
 $\frac{R_e}{a}$ - radio ecuatorial $\frac{1}{M}$ - masa de S
 $\frac{1}{a}$ - factor de calidad
 $3n$ - promedio medio

$K_d \approx 10^{-8}$. En astronomía las torcas disipativas son muy pequeñas pero actúan por tiempos muy largos.

A.C.M. Correia, J. Laskar, "Mercury's capture into $3/2$ spin-orbit resonance as a result of its chaotic dynamics," *Nature* 429, 818-820 (2007)

Tomar promedios de $L(e, t), N(e, t)$

$$\tau = \overline{\tau}(\dot{x}) = -K_d [\overline{L}(e)\dot{x} - \overline{N}(e)]$$

Tarea Repetir la derivación del mapeo estándar pero con las torcas promediadas

$$\dot{r} = \Gamma$$

$$\dot{\Gamma} = -\varepsilon W(\frac{p}{a}, e) \sin 2\chi - K_d [\overline{L}(e)\dot{x} - \overline{N}(e)]$$

Sin disipación \rightarrow mapeo estándar

Con disipación \rightarrow deducir el mapeo correspondiente.