

$$\ddot{x} = -\varepsilon \left(\frac{a}{r(t)}\right)^3 \sin(2x - 2f(t))$$

$$\dot{x} = y$$

$$\dot{y} = -\varepsilon \left(\frac{a}{r(t)}\right)^3 \sin(2x - 2f(t))$$

Kepler

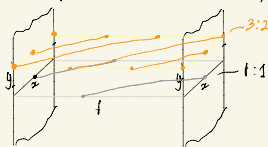
$$r(t) = a(1 - e \cos u)$$

$$f(t) = 2a \arctan \left(\sqrt{\frac{1+e}{1-e}} \tan \frac{u}{2} \right)$$

$$l_0 = l - e \sin u$$

Sección de Poincaré

(Mapa de primer retorno)



$$\mathcal{S}^1 \times \mathcal{S}^1 \times \mathbb{R}$$

$$\mathbb{T}^2 \times \mathbb{R}$$



$$\dot{x} = y$$

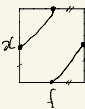
$$\dot{y} = F(x, t)$$

$$\dot{t} = 1$$

$$x \in \mathcal{S}^1, z \in \mathbb{R}$$

$$f \in \mathcal{S}$$

Luna

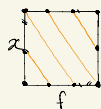


Resonancia

1:1



Mercurio

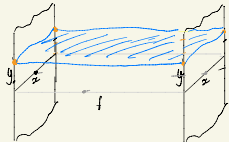
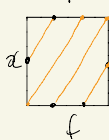


Resonancia

3:2

2 vueltas a f

3 vueltas a x



Kepler 3141



¿qué pasaría si $\varepsilon \in \mathbb{R} \setminus \mathbb{Q}$?

Tendríamos una trayectoria que llenaría el toro.

- Órbita Quasi-periódica

Esta superficie es un toro \mathbb{T}^2
 \rightarrow Este es un toro invariante del espacio fase.

$$r(t) = a(1 - e \cos u)$$

$$f(t) = 2a \arctan \left(\sqrt{\frac{1+e}{1-e}} \tan \frac{u}{2} \right)$$

$$l_0 = l - e \sin u$$

Si $e=0$ (círculo), en este caso

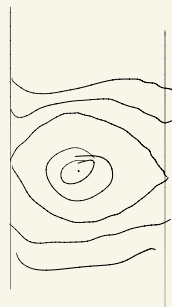
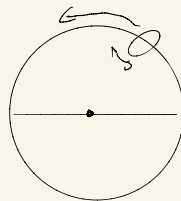
$$r = a$$

$$f = t - t_0$$

$$\ddot{x} + \varepsilon \sin(2x - 2t - 2t_0) = 0$$

$$\theta = 2x - 2t - 2t_0$$

$\ddot{\theta} + \varepsilon \sin(\theta) = 0$ ¡Esto es el péndulo!



En el caso integrable

