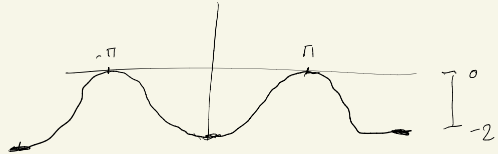
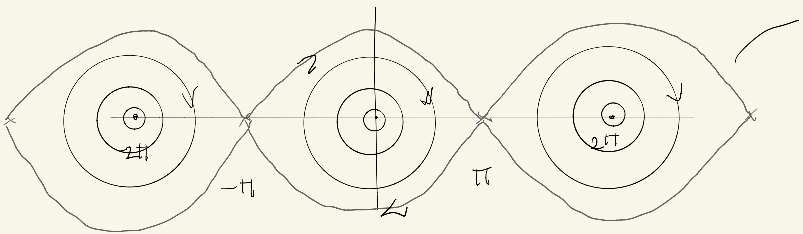
Sobre la energía

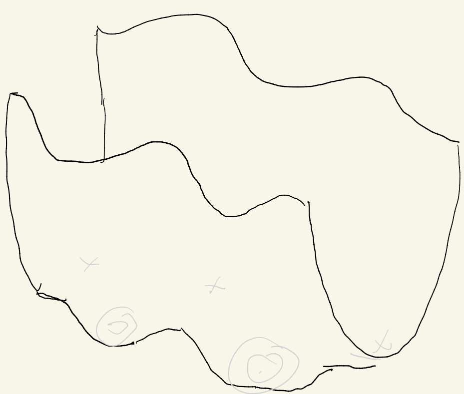
$$E = \frac{1}{2}v^2 + U(\theta)$$





Se llama separatiz

Tarda tiempo infinito de $-\pi$, π. y la integral del tiempo diverge logaritímicamente cerca de π y $-\pi$.



$E=0$

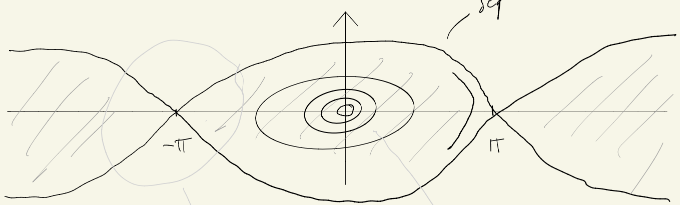
$$\frac{1}{2}v^2 + U(\theta) = 0 \iff v = \pm\sqrt{-2U(\theta)}$$

$$v = \pm\sqrt{-2(-1-\cos\theta)}$$

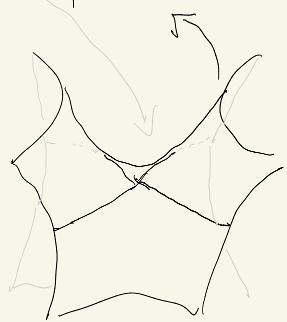
$v=0$ $\quad \sqrt{-2(-1-\cos\theta)} = 0$

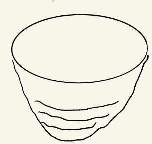
$\cos\theta = -1 \qquad \theta = (2k-1)\pi, \; k\in\mathbb{Z}$

separatrices.



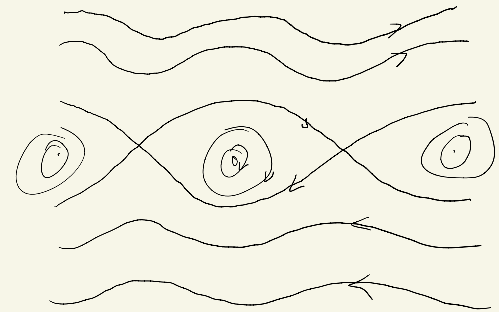
En la superficie.





$E>0$

$$v = \pm\sqrt{2(E-U(\theta))}$$



Movimiento de libración.

Linealización

$\dot\theta = v$

$\dot v = -\sin\theta$

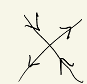
$$\begin{pmatrix}\dot\theta\\ \dot v\end{pmatrix} = f\begin{pmatrix}\theta\\ v\end{pmatrix} = \cancel{f\begin{pmatrix}\theta^*\\ v^*\end{pmatrix}}^{\,0} + \underline{Df\begin{pmatrix}\theta^*\\ v^*\end{pmatrix}\begin{pmatrix}\theta\\ v\end{pmatrix}} + \text{t.o.m.}$$

equilibrio $\Rightarrow f\begin{pmatrix}\theta^*\\ v^*\end{pmatrix} = 0$

$$Df\begin{pmatrix}\theta^*\\ v^*\end{pmatrix} = \begin{pmatrix}0 & 1\\ -\cos\theta^* & 0\end{pmatrix} \Rightarrow \begin{pmatrix}\dot\theta\\ \dot v\end{pmatrix} = \begin{pmatrix}0 & 1\\ -\cos\theta^* & 0\end{pmatrix}\begin{pmatrix}\theta\\ v\end{pmatrix}$$

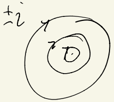
Caso $\theta^* = \pi$

$\begin{pmatrix}0 & 1\\ 1 & 0\end{pmatrix} \quad \lambda = \pm 1$



Caso $\theta^* = 0$

$\begin{pmatrix}0 & 1\\ -1 & 0\end{pmatrix} \quad \lambda = \pm i$



Teorema de Grobman-Hartman