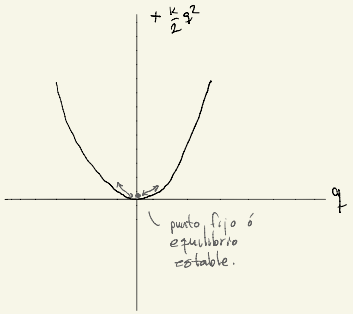
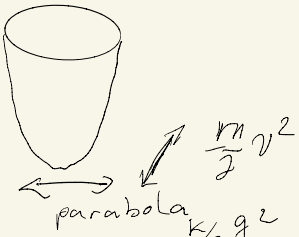
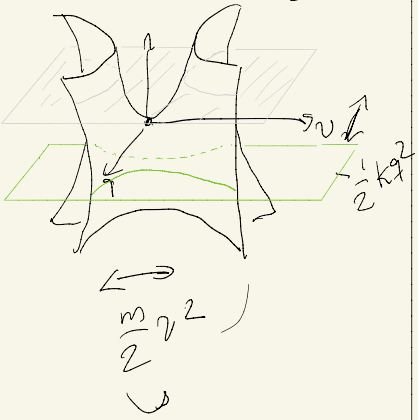


Energía $\mathbb{R}^2 \to \mathbb{R}$

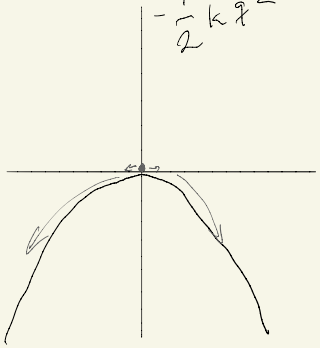




Ejemplo 2

$$u = -\frac{1}{2}kq^2$$

$$E = \frac{1}{2}mv^2 - \frac{1}{2}kq^2$$



$$E(q_0, v_0) = E = \frac{1}{2}mv^2 - \frac{1}{2}kq^2 \quad \text{hipérbolas}$$

$$(\nabla E)(q, v) = (-kq, mv)$$

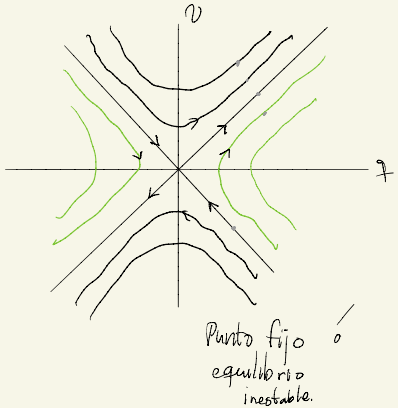
En el origen $E = 0$

$$\frac{1}{2}mv^2 = \frac{1}{2}kq^2$$

$$v^2 = \frac{k}{m}q^2$$

$$v = \pm\sqrt{\frac{k}{m}}\,q$$

dos rectas.



$E > 0$

$$v^2 = \frac{2E + kq^2}{m}$$

$$v = \pm\sqrt{\frac{1}{m}(2E + kq^2)}$$

$E < 0$

$$q^2 = \frac{1}{k}(mv^2 - 2E)$$

$$q = \pm\sqrt{\frac{1}{k}(mv^2 - 2E)}$$

$m\ddot{q} = kq$

proponemos $e^{\lambda t}$

$m\lambda^2 - k = 0 \quad , \lambda = \pm\sqrt{k/m}$

$q(t) = C_1 e^{\sqrt{k/m}\,t} + C_2 e^{-\sqrt{k/m}\,t}$

$(\dot{q}(t) = v(t)) = C_1\sqrt{k/m}\, e^{\sqrt{k/m}\,t} - C_2\sqrt{k/m}\, e^{-\sqrt{k/m}\,t}$

$q(0) = q_0 \quad , \quad v(0) = v_0$

$$C_1 = \frac{1}{2}\left(q_0 + \sqrt{\frac{m}{k}}\, v_0\right) \quad , \quad C_2 = \frac{1}{2}\left(q_0 - \sqrt{\frac{m}{k}}\, v_0\right)$$

Tiempo

$$E = \frac{1}{2}\dot{q}^2 + U(q) = \text{constante}.$$

$$\dot{q} = \pm\sqrt{2\left[E - U(q)\right]}$$

$$\frac{dq}{dt} = \sqrt{2\left[E - U(q)\right]}$$

$$dt = \frac{dq}{\sqrt{2\left[E - U(q)\right]}} \qquad \text{desde } q_0 \text{ hasta } q$$

$$t = \int_{q_0}^{q} \frac{ds}{\sqrt{2\left[E - U(s)\right]}}$$