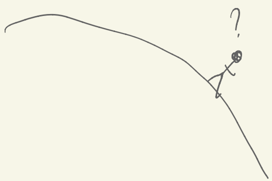
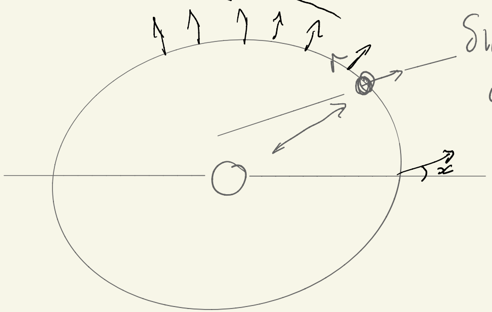
Sistema conservatio

$$m\ddot{\vec{r}} = -\nabla U(\vec{r})$$

¿Porqué la luna siempre muestra la misma cara hacia la tierra?



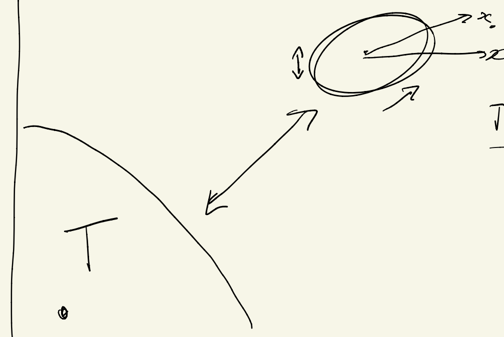
Sistema de 2 cuerpos



Sistema de órbita-espín.

$$m\ddot{x} = -\nabla U(x)$$

Peale



$$m\ddot{x} = -\nabla U(x) - \alpha\dot{x}$$

Torcas de Marea.

Sistemas de Partículas

Partículas $1, \dots, s$
masas $m_1, \dots, m_s$
posiciones $\vec{r}_1, \dots, \vec{r}_s \in \mathbb{R}^N$, $N = 1,2,3$

$$m_i \ddot{r}_i = F_i(\vec{r}_1, \dots, \vec{r}_s)$$

La fuerza depende de la posición de cada partícula.

Sistema conservativo de s partículas

$$m_i \ddot{\vec{r}}_i = -\nabla_{\vec{r}_i} U(\vec{r}_1, \dots, \vec{r}_s)$$

$\nabla_{\vec{r}_i}$: gradiente respecto a $r_i = (x_i, y_i, z_i)$

en $\mathbb{R}^3$

$$\nabla_{\vec{r}_i} = \left(\frac{\partial}{\partial x_i}, \frac{\partial}{\partial y_i}, \frac{\partial}{\partial z_i}\right)$$

Energía

$$E = \underbrace{\sum_{i=1}^{s} \left(\frac{1}{2} m_i |\dot{\vec{r}}_i|^2\right)}_{\text{energía cinética}} + \underbrace{U(\vec{r}_1, \dots, \vec{r}_s)}_{\text{energía potencial}}$$

$$|\dot{\vec{r}}_i|^2 = \langle \dot{\vec{r}}_i, \dot{\vec{r}}_i \rangle$$

$$\frac{d}{dt} E = \sum_{i=1}^{s} \frac{1}{\cancel{2}} m_i \cancel{2} \langle \dot{\vec{r}}_i, \ddot{\vec{r}}_i \rangle + \sum_{i=1}^{s} \langle \nabla_{\vec{r}_i} U(\ ), \dot{\vec{r}}_i \rangle$$

$$= \sum_{i=1}^{s} \left(\langle \dot{\vec{r}}_i, -\nabla_{r_i} U(\ ) \rangle + \langle \nabla_{\vec{r}_i} U(\ ), \dot{\vec{r}}_i \rangle \right)$$

$$= 0.$$

en $\mathbb{R}^3$

$$\vec{r} = (\vec{r}_1, \dots, \vec{r}_s) = [x_1, y_1, z_1, \dots, x_s, y_s, z_s] \in \mathbb{R}^{3s}$$

$$\dot{\vec{r}}$$

$$\nabla = [\nabla_{\vec{r}_1}, \dots, \nabla_{\vec{r}_s}] = \left[\frac{\partial}{\partial x_1}, \frac{\partial}{\partial y_1}, \frac{\partial}{\partial z_1}, \dots \right]$$

$$M = \begin{bmatrix} \begin{matrix} m_1 & 0 & 0 \\ 0 & m_1 & 0 \\ 0 & 0 & m_1 \end{matrix} & & & \\ & \begin{matrix} m_2 & 0 & 0 \\ 0 & m_2 & 0 \\ 0 & 0 & m_2 \end{matrix} & & \\ & & \ddots & \\ & & & \begin{matrix} m_s & 0 & 0 \\ 0 & m_s & 0 \\ 0 & 0 & m_s \end{matrix} \end{bmatrix}$$

Ecuaciones de Newton

$$M \ddot{\vec{r}} = -\nabla U(\vec{r})$$

$$E = \frac{1}{2} \langle \dot{\vec{r}}, M \dot{r} \rangle + U(\vec{r})$$

$$\frac{dE}{dt} = 0$$